Matheseos Universalis

formulas fundamentales

(Allmänna Storhetslärans grundformler)

Præside

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§1. Jussu legum Acad. dissertationem a me editam pro laurea publice defensuro formulæ illæ fundamentales Math. universalis, quas prælegente doctissimo præside collegi et nuntium Matheseos illius universalis, quam meditatur, censui, præ ceteris mihi arridebant. Ad methodum illam plane novam, quam in principiis Matheseos firmandis adhibet Præses, jamjam antea pluribus in locis animos attendit doctorum ex. gr. in Scandia (X. 1) "inledning till läran om gentagna functioner," in disquisitione illa "prolegomena till hvarje blifvande allmän storhetslära," quam in coetu eruditorum Gotheburgensi exhibuit, denique in fine regularum hocce anno editarum derivandi et differentiandi generalium. Notione summæ vel aggregati, quam quidem notionem hucusque haud probe explicatam habemus, his in scriptis posterioribus dialectice explorata, nos docet auctor, hancce notionem latiorem majoremque habere sensum, et quidem *summam* ea, quæ sequitur, formula definiri posse: $(x|y)|z=(z|y)|x$, cui hæc jungatur $x|J=x$, in qua J significat quantitatem hac ipsa æquatione ita constitutam, ut x quamcumque quantitatem significet. Quo in loco signum $|$, quod nominamus "magis", non solum $+$ (plus) sed etiam uniuscujusque, quæ ejusmodi est, ut æquationes nuper allatæ, quæcunque sint quantitates illæ x, y, z, valeant, operationis signum esse potest. Quoties igitur formula $x|y$ utimur, ejusmodi aggregatum *) quantitatum x & y intelligimus. Hac vero formula $x|y|z$ promiscue $(x|y)|z$ & $x|(y|z)$ intelligimus, aggregatumque trium x, y & z nuncupamus, & similiter plurium aggregatum definitur. Si autem $x|y=z$ ponitur, dicimus x esse **) *residuum* ipsorum z & y, quod ita exprimitur: $x=z\lceil y$ vel $z\lceil y$. Si jam $y=J$, quod quidem J vocamus *adiaphoron* (indifferent), ponimus, facile tibi erit demonstrare, hancce summam esse functionem partium suarum symmetricam sc: $x|z=z|x$. Quinimo tibi facile persuadeas, differentiam illam formalem $x\lceil y$ ut summam quantitatis x et alius cujusdam u vel $\overline{y}$, quam y *negativum* vocamus, haberi posse. De his vero formis valent *axiomata* 1) Si $x|y=z=y|x$ fuerit, erit $x=z\lceil y$ & $y=z\lceil x$; 2) Sin vero $z\lceil y=x$ fuerit, erit $z=x|y$. 3) Si $x=y$ fuerit, erit $x|z=y|z$ & $x\lceil z=y\lceil z$. Ex quibus facile probantur omnes formulæ fundamentales *additionis* et *subtractionis*, tum illius vulgaris tum hujus amplioris et formalis, quarum definitio ex illis, quæ supra attulimus, facile elicitur. Quarum quidem formularum jam exempla exhibet Tab. 1:ma. Observandum tamen est, quamcumque formulam additionis et substractionis vulgaris,

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*) Suetice redditum: allmän eller formell summa.

**) Suetice redditum: allmän eller formell skillnad.

Tab. 1	Binomium				Trinomium Universale	Negativum
	Aggregatum Allm. Summa	Residuum Allm. Skilnad	Adiaphoron Indifferent	Formula indifferentis	Allm. Summa af 3 storheter (at addera en Summa)	Allm. neg. storhet.
Ex.	$a\|b=c=b\|a$	$b=c\lceil a=c\lceil a$	$a\lceil a=O=J$	$O\|c=c=c\|o=c\lceil O$	$a\|(b\|c)=a\|b\|c=(a\|b)\|c=(c\|b)\|a$	$\overline{X}=O\lceil X$
Summa	$a+b=c=b+a$	$b=c-a$	$a-a=o$	$o+c=c=c\pm o$	$a+(b+c)=a+b+c=(a+b)+c$	$-X=o-X$
Product	$a.b=c=b.a$	$b=c:a=\frac{c}{a}$	$a:a=1$	$1.c=c=c.1=c:1$	$a.(b.c)=a.b.c=(a.b).c$	$\overline{X}=1:X=\frac{1}{X}=\overline{X}$
Signum	$ab_\eta=c=ba_\eta$	$b=\frac{c}{a}:\eta$	$1:\eta=J$	$Jc_\eta=c=cJ_\eta$	$a(bc\eta)\eta=(ab\eta)c\eta=(c\|b)a.\eta=$ $=(c.b.\eta).a.\eta$	$\overline{X}=J:X_\eta=1:X_\eta$
a♁b=	$a+b+nab=c$ (= a♁b)	$b=\frac{c-a}{1+na}$	o	$c+o(1+nc)=c$	$c+\overline{1+nc}.(a+b+nab)=$ $=a+\overline{1+na}.(b+c+nbc)$	$\frac{-X}{1+nX}$
a♀b	$\frac{a+b+nab}{1+mab}=c$	$b=\frac{c-a}{1+na-mca}$ $=c$♀$\overline{a}$	o	$\frac{c+o(1+nc)}{1+mc.o}=c$	$\frac{a+(1+na).\frac{b+c+nbc}{1+mbc}}{1+ma.\frac{b+c+nbc}{1+mbc}}=$ $=\frac{c+(1+nc).\frac{a+b+nab}{1+mab}}{1+mc.\frac{a+b+nab}{1+mab}}$	$(\overline{X}=)\frac{-X}{1+nX}$

Regulæ jungendi & deducendi.

Aggregatum deducere At subtrahera en summa.	Residuum addere vel deducere At addera eller Subtrahera en skilnad.	Residua addere At addera skillnader.
$a\lceil(b\vert c)=(a\lceil b)\lceil c$	$(a\vert b)\lceil c=a\vert(b\lceil c)=a\lceil(c\lceil b)$	$(a\lceil b)\vert(c\lceil d)=(a\vert c)\lceil(b\vert d)$
$a-(b+c)=(a-b)-c$	$(a+b)-c=a+(b-c)=a-(c-b)$	$(a-b)+(c-d)=(a+c)-(b+d)$
$a:(b.c)=(a:b):c=\frac{a}{b}:c$	$(a.b):c=a.(b:c)=a:(c:b)=\frac{a}{(\frac{c}{b})}$	$\frac{a}{b}.\frac{c}{d}=\frac{a.c}{b.d}$
$a:(bc\eta)\eta=(a:b\eta):c\eta$	$(ab\eta):c\eta=a(b:c\eta)\eta=a:(c:b\eta)\eta\left(=\frac{ab}{c}\right)$	$\left(\frac{a}{b\eta}\right).\left(\frac{c}{d\eta}\right).\eta=ac\eta:(bd\eta)\eta$
$\frac{a-(b+c+nbc)}{1+n(b+c+nbc)}=\frac{\frac{a-b}{1+nb}-c}{1+nc}$	$\frac{(a+b+nab)-c}{1+nc}=a+(1+na).\frac{b-c}{1+nc}=\frac{a-\frac{c-b}{1+nb}}{1+n.\frac{c-b}{1+nb}}$	$\frac{a-b}{1+nb}+\frac{c-d}{1+nd}+n.\frac{a-b}{1+nb}.\frac{c-d}{1+nd}=\frac{a+c+nac-(b+d+nbd)}{1+n(b+d+nbd)}$
$\frac{a-\frac{b+c+nbc}{1+mbc}}{1+(n-ma)\frac{b+c+nbc}{1+mbc}}=\frac{\frac{a-b}{1-ma+nb}-c}{1+nc-m.\frac{a-b}{1-ma+nb}.c}$	$\frac{(a\text{♀}b)-c}{\left\{\begin{matrix}1+nc\\-m.(a\text{♀}b).c\end{matrix}\right\}}=\frac{a+\overline{1+na}.\frac{b-c}{1+nc-mbc}}{1+ma\frac{b-c}{1+nc-mbc}}=\frac{a-\frac{c-b}{1+nb-mbc}}{1+n.\frac{c-b}{1+nb-mcb}-ma(c\overline{\text{♀}}b)}$	$\frac{\left\{\frac{a-b}{1+\overline{n-mab}}+\frac{c-d}{1+\overline{n-mcd}}+\frac{n.(a-b)(c-d)}{(1+\overline{n-mab})(1+\overline{n-mcd})}\right\}}{1+m.\frac{(a-b)(c-d)}{(1+\overline{n-mab})(1+\overline{n-mcd})}}=\frac{\frac{a+c+nac}{1+mac}-\frac{b+d+nbd}{1+mbd}}{1+\left(n-m.\frac{a+c+nac}{1+mac}\right)\frac{b+d+nbd}{1+mbd}}$

Tabula Secunda	Multipl. aggregat.	allmän product	Multiplum aggregati — 2 & 3. Theor. at multiplicera en Summa
	$a\vert b$	$r\odot a$	$n\odot(a\vert\int b)=(n\odot a)\vert\int(n\odot b)$
	$a+b$	$r.a$	$n.(a\pm b)=(n.a)\pm(n.b)$
	$a.b$	a^{r}	$(a.b)^{n}=a^{n}b^{n}$ & $(a:b)^{n}=a^{n}:b^{n}$
	$a+b+\gamma ab$ $=a\text{♁}b$	$\frac{\overline{1+\gamma a}^{r}-1}{\gamma}$	$\frac{1}{\gamma}.(1+\gamma(a+b+\gamma ab))^{n}-\frac{1}{\gamma}=(n\odot a)\text{♁}(n\odot b)$
			$=\overline{n\odot a}+\overline{n\odot b}+\gamma.\overline{n\odot a}.\overline{n\odot b}$
			$\frac{1}{\gamma}.\left(1+\gamma.\frac{a-b}{1+\gamma b}\right)^{n}-\frac{1}{\gamma}=\frac{\overline{n\odot a}-\overline{n\odot b}}{1+\gamma(n\odot b)}$
	$\frac{a+b}{1-ab}$	$T\overline{r\perp a}$	$T\left(n\perp\frac{a+b}{1-ab}\right)=\frac{T\overline{n\perp a}+T\overline{n\perp b}}{1-T\overline{n\perp a}\cdot T\overline{n\perp b}}$
			$Tn\perp\left(\frac{a-b}{1+ab}\right)=\frac{T\overline{n\perp a}-T\overline{n\perp b}}{1+T\overline{n\perp a}.T\overline{n\perp b}}$

positis signis magis (|) et minus formali (⌈ vel ʃ) pro signis plus (+) & minus (—), universalem reddi poss tali modo ex formula vulgata: $a-(b+c)=(a-b)-c$ consequimur illam generalem: $a\int(b\vert c)=(a\int b)\int c$, quæ in innumeras alias continet, et rursum specialis reddita nobis tradit: $a\text{♀}(b\text{♀}c)=(a\text{♀}\overline{b})\text{♀}\overline{c}$, in qua ex. grat. $a\text{♀}b$ signi cat $a+b+nab$ et igitur $c\text{♀}\overline{a}=\frac{c-a}{1+na}$; quam si signorum significationem adhibemus, a simplici illa formula superio progredimur ad illam: $\frac{a-(b+c+nbc)}{1+n(b+c+nbc)}=\frac{\frac{a-b}{1+nb}-c}{1+nc}$, cujus probatio eadem est ac superioris seu in hac ipsa latet, præterea, si lubet, algebraice explicari potest.

§ 2. Additioni unicuique respondet sua multiplicatio ita, ut $a\vert a\vert a\,..$ vel aggregatum plurium a æqualiu sit *multiplum* formale ejusdem a, quod exprimitur: $n\odot a$. Si autem e contrario ponimus $n\odot a=b$, dicimus a e

multiplum multipli	aggregatum multiplorum
at multiplicera formella producter	4 & 5 Theor. at addera producter
$n\odot\overline{r\odot a}=r\odot\overline{n\odot a}=(n.r)\odot a$	$(n\odot a)\,\vert\lceil(r\odot a)=(n\pm r)\odot a$
$n.(r.a)\;=r.(n.a)\;=(n.r).a$ $(a^r)^n\;=(a^n)^r\;=a^{(nr)}$	$\overline{n.a}\pm\overline{r.a}\;=\overline{n\pm r}.a$ $a^n.a^r\;=a^{n+r}$ & $a^n:a^r=a^{n-r}$
$\frac{1}{\gamma}.\left(1+\gamma.\frac{\overline{1+\gamma a}^{n}-1}{\gamma}\right)^{r}-\frac{1}{\gamma}=$ $=\frac{1}{\gamma}.\left(1+\gamma.\frac{\overline{1+\gamma a}^{r}-1}{\gamma}\right)^{n}-\frac{1}{\gamma}=\frac{1}{\gamma}(\overline{1+\gamma a}^{\,(nr)}-1)$	$\frac{\overline{1+\gamma a}^{n+r}-1}{\gamma}=n\odot a+r\odot a+\gamma.\overline{n\odot a}.\overline{r\odot a}$ $\frac{\overline{1+\gamma a}^{n-r}-1}{\gamma}=\frac{\overline{n\odot a}-\overline{r\odot a}}{1+\gamma.\overline{r\odot a}.}$
$Tn\perp(Tr\perp x)=Tr\perp(Tn\perp x)=T(\overline{n.r}\perp x)$	$\frac{\overline{Tn\perp a}\pm\overline{Tr\perp a}}{1\mp\overline{Tn\perp a}.\overline{Tr\perp a}}=T(n\pm r)\perp a$

quotum 1. *partem formalem* quantitatis b, et n esse *exponemtem rationis* inter b et a, quod exprimitur: $n=b\odot\overrightarrow{a}$. Ut autem in Cap. D doctrinæ proportionum, quam edidit Præses, sic etiam hoc in loco sententiæ hæ de multiplicatione formali demonstrantur:

Th. I. Si $a=b$; tunc: $n\odot a=n\odot b$.

Th. II. $n\odot(a\vert b)=\overline{n\odot a}\vert\overline{n\odot b}$.

Cor. $n\odot(r\odot a)=r\odot(n\odot a)$

Th. III. $n\odot(a\lceil b)=\overline{n\odot a}\lceil\overline{n\odot b}$.

Th. IV. $\overline{n\odot a}\vert\overline{r\odot a}=(n+r)\odot a$.

Cor. $n\odot(r\odot a)=(n.r)\odot a$

Th. V. $\overline{n\odot a}\lceil(r\odot a)=(n-r)\odot a$.

Quarum sententiarum exempla exhibet Tab. II.

Tab. III	aggregatum	quoti, aliquoter	quotus aggregati. at dividera summor.	quotus multipli. at dividera producter
	$a \mid b$	$\overline{r} \odot a$	$\overline{n} \odot (a \mid b) = (\overline{n} \odot a) \mid (\overline{n} \odot b)$	$\overline{n} \odot (r \odot a) = r \odot (\overline{n} \odot a) = \frac{r}{n} \odot a$
	$a + b$	$\frac{a}{r}$	$\frac{a+b}{n} = \frac{a}{n} + \frac{b}{n}$	$\frac{r.a}{n} = r.\frac{a}{n} = \frac{r}{n}.a$
	$a.b$	$\sqrt[r]{a}$	$\sqrt[n]{a.b} = \sqrt[n]{a}.\sqrt[n]{b}$ & $\sqrt[n]{(a:b)} = \sqrt[n]{a}:\sqrt[n]{b}$	$\sqrt[n]{(a^r)} = \left(\sqrt[n]{a}\right)^r = a^{r:n}$
	$a+b+\gamma ab$	$\frac{\overline{1+\gamma a}^{\frac{1}{r}} - 1}{\gamma}$	$\frac{\overline{1+\gamma(a+b+\gamma ab)}^{\frac{1}{n}} - 1}{\gamma} = \frac{\overline{1+\gamma a}^{\frac{1}{n}} - 1}{\gamma} \mid \frac{\overline{1+\gamma b}^{\frac{1}{n}} - 1}{\gamma}$ et $\overline{n} \odot \left(\frac{a-b}{1+\gamma b}\right) = \frac{(\overline{n} \odot a) - (\overline{n} \odot b)}{1+\gamma.(\overline{n} \odot b)}$	$\frac{1}{\gamma}\left(1+\gamma.\frac{\overline{1+\gamma a}^{r} - 1}{\gamma}\right)^{\frac{1}{n}} - \frac{1}{\gamma} = \frac{1}{\gamma}.\left(1+\gamma\frac{\overline{1+\gamma a}^{\frac{1}{n}} - 1}{\gamma}\right)^{r} - \frac{1}{\gamma}$
	$\frac{a+b}{1-ab}$	$T\left(\frac{1}{r}.L a\right)$	$T\left(\frac{1}{r}.L \frac{a \pm b}{1 \mp ab}\right) = \frac{T\left(\frac{1}{r}.L a\right) \pm T\left(\frac{1}{r}.L b\right)}{1 \mp T\left(\frac{1}{r}.L a\right) \perp \left(\frac{1}{r}.L b\right)}$	$T\frac{1}{n}(\perp T r.L a) = T r.L T \frac{1}{n} \overline{L a}$

Tab. IV:a	aggregatum	Logarithmus	Analogia aggregati et differentiæ
	$\frac{a \mid b}{a+b}$	$\alpha \odot \overline{b} = \Lambda a : \Lambda b$ $a : b$	$\alpha \odot \overline{a} = \beta \odot \overline{b}; = (\alpha \mid \beta) \odot \overline{(a \mid b)} = (r \odot \alpha) \odot \overline{(r \odot a)} = (\alpha \lceil \beta) \odot \overline{(a \lceil b)}$ $\alpha : a = \beta : b; = (\alpha + \beta) : (a + b) = (r.\alpha) : (r.a) = (\alpha - \beta) : (a - b)$
	$a.b$	$\frac{\alpha}{b} = \frac{a}{b} = \frac{La}{Lb}$	$\frac{\alpha}{a} = \frac{\beta}{b}; = \frac{\alpha.\beta}{a.b} = \frac{\alpha^r}{a^r} = \frac{\alpha:\beta}{a:b}:$ & $\frac{\alpha}{a}.\frac{a}{c} = \frac{\alpha}{c}$
	$a+b+\gamma ab$	$\frac{1+a\gamma}{1+b\gamma}$	$\frac{1+\alpha\gamma}{1+\gamma a} = \frac{1+\gamma\beta}{1+b\gamma}; = \frac{1+\gamma(\alpha+\beta+\gamma\alpha\beta)}{1+\gamma(a+b+\gamma ab)} = \frac{(\overline{1+\gamma\alpha}^r - 1):\gamma}{(\overline{1+\gamma a}^r - 1):\gamma} = \frac{\left(1+\gamma.\frac{\alpha-\beta}{1+\gamma\beta}\right)}{1+\gamma\left(\frac{a-b}{1+\gamma b}\right)}$
	$\frac{a+b}{1-ab}$	$\perp a : \perp b$	$L\alpha : \perp a = L\beta : L b; = L\frac{\alpha+\beta}{1-\alpha\beta} : L\frac{a+b}{1-ab} = L\,\overline{T r L \alpha} : L\,\overline{T r \perp a} = L\frac{\alpha-\beta}{1+\alpha\beta} : L\frac{a-b}{1+ab}$

Analogiæ

Hypothesis — **Thesis**

$$\text{Om } \overline{n}\odot\alpha=\overline{r}\odot\beta;\ \overset{\text{så}}{=}\ \overline{(n+r)}\odot(\alpha|\beta)=\overline{(n-r)}\odot(\alpha\lceil\beta)=\overline{(r.n)}\odot(r.\odot\alpha)=\frac{1}{n}\odot\alpha$$

$$\frac{\alpha}{n}=\frac{\beta}{r};=\frac{\alpha+\beta}{n+r}=\frac{\alpha-\beta}{n-r}=\frac{1}{n}.\alpha=\frac{r.\alpha}{r.n}$$

$$\sqrt[n]{\alpha}=\sqrt[r]{\beta};=\sqrt[n+r]{(\alpha.\beta)}=\sqrt[n-r]{(\alpha:\beta)}=\sqrt[rn]{(\alpha^{r})}=\alpha^{\frac{1}{n}}$$

$$\overbrace{1+\gamma\alpha}^{\frac{1}{n}}=\overbrace{1+\gamma\beta}^{\frac{1}{r}};=(1+\gamma(\alpha+\beta+\gamma\alpha\beta))^{\frac{1}{n+r}}=\left(1+\gamma.\frac{\alpha-\beta}{1+\gamma\beta}\right)^{\frac{1}{n-r}}=\left(1+\gamma\frac{\overbrace{1+\gamma\alpha}^{r}-1}{1}\right)^{\frac{1}{rn}}$$

$$T\frac{1}{n}\perp\alpha=T\frac{1}{r}\perp\beta;=T\overbrace{\frac{1}{n+r}\perp\frac{\alpha+\beta}{1-\alpha\beta}}=T\overbrace{\frac{1}{n-r}\perp\frac{\alpha-\beta}{1+\alpha\beta}}=T\left(\frac{1}{rn}\perp(T\overbrace{r\perp\alpha})\right)$$

Logarithmorum additio	Logarithmus producti
$(\alpha\overline{\odot a})\pm(\beta\overline{\odot a})=(\alpha\|\lceil\beta)\overline{\odot a}$ seu $A(\alpha\|\beta)=A\alpha+A\beta$	$(r\odot\alpha)\overline{\odot a}=r.(\alpha\overline{\odot a})$ & $r\odot\alpha=(r.(\alpha\overline{\odot a}))\odot a$
$(\alpha:a)\pm(\beta:a)=(\alpha\pm\beta):a$	$(r.\alpha):a=r.(\alpha:a)$ & $r.\alpha=(r.(\alpha:a)).a$
$\underbrace{\frac{\alpha}{a}}+\underbrace{\frac{\beta}{a}}=\underbrace{\frac{\alpha.\beta}{a}}$ & $L\alpha-L\beta=L(\alpha:\beta)$	$\underbrace{\frac{\alpha^{r}}{a}}=r.\underbrace{\frac{\alpha}{a}}$ & $\alpha^{r}=\alpha^{r.\underbrace{\frac{\alpha}{a}}}$
$\underbrace{\frac{1+\alpha\gamma}{1+a\gamma}}+\underbrace{\frac{1+\beta\gamma}{1+a\gamma}}=\underbrace{\frac{1+\gamma(\alpha+\beta+\gamma\alpha\beta)}{1+a\gamma}}$	$L(\alpha^{r})=r.L\alpha$
$\underbrace{\frac{1+\alpha\gamma}{1+a\gamma}}-\underbrace{\frac{1+\beta\gamma}{1+a\gamma}}=\underbrace{\frac{1+\gamma\frac{\alpha-\beta}{1+\gamma\beta}}{1+a\gamma}}$	$\underbrace{\frac{(\overbrace{1+\gamma\alpha}^{r}-1):\gamma}{1+a\gamma}}=r.\frac{1+\gamma\alpha}{1+\gamma a}$
$(\perp\alpha:\perp a)+(\perp\beta:\perp a)=\perp\frac{\alpha+\beta}{1-\alpha\beta}:\perp a$	$\perp Tr\perp\alpha:\perp a=r.(\perp\alpha:\perp a)$
$\perp a-\perp\beta=\perp\frac{\alpha-\beta}{1+a\beta}$	

§ 3. Ex iis, quæ allata sunt de multiplo, sententiæ illis congruæ de *partibus form.* facile deducuntur; posi enim: $n\odot a=\alpha$ et $n\odot b=\beta$, habemus $a=\overline{n\odot}\alpha$ et $b=\overline{n\odot}\beta$.

Unde Th. I. Si $\alpha=\beta$, habemus $\overline{n\odot}\alpha=\overline{n\odot}\beta$ vel saltem $\overline{n\odot}\alpha=(\overline{n\odot}\beta)\,|\,(\overline{n\odot}J)$.

Th. II. $\overline{n\odot}(a|b)=(\overline{n\odot}a)|(\overline{n\odot}b)|(\overline{n\odot}J)$

Cor. $\overline{n\odot}(r\odot a)=r\odot(\overline{n\odot}a)|(\overline{n\odot}J)$, et $\overline{n\odot}(\overline{r\odot}a)=\overline{r\odot}(\overline{n\odot}a)|\overline{r\odot}(\overline{n\odot}J)$.

Th. III. $\overline{n\odot}(a\lceil b)=(\overline{n\odot}a)\lceil(\overline{n\odot}b)|(\overline{n\odot}J)$.

Th. IV. Si $\overline{n\odot}\alpha=\overline{r\odot}\beta$, habemus $\overline{(n+r)\odot}(\alpha|\beta)=\overline{n\odot}\alpha$; &

Th. V. $\overline{(n-r)\odot}(\alpha\lceil\beta)=\overline{n\odot}\alpha$.

Cor. (Th. IV.) $\overline{n\odot}\alpha=\overline{(r.n)\odot}(r\odot\alpha)$ et $\overline{n\odot}\alpha=\frac{1}{n}\odot\alpha$; atque $\overline{n\odot}(n\odot\alpha)=1\odot\alpha=\alpha=\overline{1\odot}\alpha$.

Exempla exhibet Tab. III.

§ 4. Posito denique $n\odot a=\alpha$ et itaque $n={}_\alpha\overline{\odot}a$ etc. sententias, quæ sequuntur, de *exponentibus ration* (seu Logarithmis) formalibus colligimus:

Th. I. Si $n\odot a=r\odot a$, habemus $n=r$, nisi $a=\overline{(n-r)}\odot J$.

Th. II. Si ${}_\alpha\overline{\odot}a={}_\beta\overline{\odot}b$, habemus $(\alpha|\beta)\overline{\odot}(a|b)={}_\alpha\overline{\odot}a$.

Cor. ${}_\alpha\overline{\odot}a=(r\odot\alpha)\overline{\odot}(r\odot a)$.

Th. III. Si ${}_\alpha\overline{\odot}a={}_\beta\overline{\odot}b$, habemus $(\alpha\lceil\beta)\overline{\odot}(a\lceil b)={}_\alpha\overline{\odot}a$.

Th. IV. $(\alpha\overline{\odot}a)+(\beta\overline{\odot}a)=(\alpha|\beta)\overline{\odot}a$.

Cor. $r.(\alpha\overline{\odot}a)=(r\odot\alpha)\overline{\odot}a$.

Th. V. $(\alpha\overline{\odot}a)-(\beta\overline{\odot}a)=(\alpha\lceil\beta)\overline{\odot}a$.

Exempla exhibet Tab. IV.
